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SIMILARITY CRITERIA FOR DUSTING OF PLASTIC MATERIALS

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We propose a system of similarity criteria and criteria formulas that have made it possible to generalize the available experimental data on the erosion of plastic materials (metals) by particles, impacting with them at a right angle, with a root-mean-square error of ~30%. The experimental data analyzed have been obtained under the following conditions: velocity of the particles, 10–300 m/sec; temperature of the obstacle, room temperature–0.8 T_m*; ratio between the densities of the particle flux and the obstacle material, 0.3–3; diameter of the particles, 40–3000 μm. A formula for generalizing experimental data on the erosion of materials by particles incident on their surface at different angles has been obtained.*

In many technical processes, materials are subjected to erosion (dusting) by solid particles impacting them, which is considered as a negative phenomenon in the majority of cases, for example, in the case of operation of helicopter propellers and blades in a dust-laden atmosphere [1]. At the same time, fluxes of solid particles can be put to good use, for example, for the treatment and cutting of rocks [2]. The problem of erosion of materials by fluxes of solid particles came under scientific scruting in the early twentieth century [3]. Since that time a large amount of factual data on this process and its features has been accumulated. It has been established that the processes of erosion of brittle and plastic materials are essentially different. Brittle materials break down as a result of the formation of cross cracks and their outcrop. Erosion tracks are similar to spallings. One particle participates in one elementary act of erosion. The highest rate of erosion of materials is observed in the case where particles are incident on their surface at a right angle. Plastic materials are broken down as a result of the formation of depressions (craters) by particles, impacting their surface at a right angle, and furrows by particles, incident at an oblique angle, and as a result of the transfer of the material to the edges of the depressions and its subsequent removal by impacts of other particles or by the carrying gas flow. In the general case, several particles participate in an elementary act of erosion. The angle of impact of particles with an obstacle at which the rate of erosion is maximum differs from a right angle.

An S-shaped dependence of the erosion of materials on the hardness H of the particles impacting them has been revealed in [4]. Erosion is practically absent at small H; then it increases sharply with increasing H, and at large H the strength characteristics of the particle material do not influence the erosion rate.

Depressions can arise in plastic materials because of their yielding. A compression of an isotropic solid material shifts its layers. If the shear stress of a material exceeds its ultimate strength, the material becomes yielding. In this case, the critical load, according to [5], is equal to

$$p_{\text{cr}} = 2 \cdot \frac{1-\nu}{1-2\nu}\,\tau_{\text{cr}}\,. \tag{1}$$

The correctness of this formula was supported by experiments [6], in which the pressure at the contact of an indenter with a material increased with increase in the load on the indentor to $3\tau_{\text{cr}}$ and then remained constant. It is easy to calculate that at $\nu = 0.25$ the coefficient of τ_{cr} in formula (1) is equal to three. Having equated the critical load to the differential pressure caused by a shock wave ($p_{\text{sh.w}} = \omega\gamma V$, $\gamma = \omega'/(\omega + \omega')$, $\omega = \rho a$, $\omega' = \rho' a'$), we can determine the critical impact velocity:

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$$V_{\mathrm{cr}} = \frac{2}{\omega\gamma}\frac{1-\nu}{1-2\nu}\tau_{\mathrm{cr}} . \tag{2}$$

The calculations have shown that V_{cr} falls within the range 35–60 m/sec for aluminum, duraluminum, copper, and steel. A plastic, deformational erosion can arise in the case where $V > V_{\mathrm{cr}}$. If $V < V_{\mathrm{cr}}$, the erosion is due to the disordering of the material by fatigue cracks.

The erosion of a material by particles represents a complex process dependent on many factors that often are accidental in character and therefore difficult to take into account. This is confirmed by the existence of numerous theoretical approaches to the solution of this problem [7–9] and, at the same time, by the absence of a theory adequate to the known experimental data. In such a situation, a good result can be obtained by generalizing experimental data with the use of similarity criteria, since almost all statements of practical aero- and hydrodynamics are almost entirely based on semiempirical formulas.

In deciding on similarity criteria, it is necessary to reveal the main parameters of the physical process considered, orienting oneself to the existing ideas of the process as well as the parameters and their combinations that have already been used for processing of experimental data. One of the most important characteristics of the process under study is the impact velocity. Many researchers generalized experimental data in the form

$$W = bV^{m} , \tag{3}$$

where b and m are usually constants for the given process and, at best, for the materials participating in it.

Another important characteristic is the strength parameter of the materials participating in the interaction. The densities of materials naturally determine their erosion rate. As is seen from formula (2), the process can also be dependent on the velocity of sound in the materials (the sound impedance). The strength of a material decreases with increase in its temperature, and the process of breakdown of the material becomes distinctly different when its surface reaches the melting temperature. It was also reported that the density of the particle flux impacting the surface of plastic materials, i.e., the concentration of particles in the carrying flux, influences the erosion of these material. The direct influence of the particle sizes on the erosion rate remains a point open to question.

Our investigations have shown that the following quantities should be considered as the main parameters of the process under study: V, ρ', a', d, α, and c_{s} for a particle and ρ, a, T, τ_{t}, and T_{m} for an obstacle. A similarity criterion to be determined is the dimensionless rate of erosion W, equal to the ratio between the erosion mass carried away from the surface of the obstacle in a unit time and the mass of particles incident on it in a unit time. The number of determining similarity criteria N is found using the π-theorem of similarity and the dimensional theory $N = i - z$, where i is the number of independent variables characterizing the phenomenon considered and z is the number of their primary dimensions. In the case under study, the primary dimensions are length, mass, time, and temperature. Thus, the total number of determining similarity criteria should be equal to seven.

The problem considered was investigated with the use of statistical criteria analysis for the first time in [7]. In that work, V, α, H, B, R, and E were used as the determining parameters. The last-named parameter was determined in terms of the hardness and strength of the obstacle material. It is seen that this system of determining parameters does not involve characteristics of the dynamic interaction of particles with an obstacle. Moreover, one and the same system of determining parameters was used for brittle and plastic materials. This approach does not correspond to the modern level of knowledge of the process considered. A criteria generalization of experiments on the erosion of brittle materials has been performed in [8]; however, the formulas obtained in that work did not have an independent meaning and were used only for comparison of the erosion theories developed in [9].

Similarity criteria — dimensionless combinations of determining parameters — are independent if none of them can be obtained from the other parameters through a logarithmically linear combination. Since operations of production of similarity criteria or raising of them to any finite power also give similarity criteria, the number of sets of determining (and to be determined) similarity criteria consisting of N complexes is infinitely large. It is common to find a set that would be the most simple and would reflect, as fully as possible, the physical features of the phenomenon at the current level of knowledge of it.

It is thought that the process of impact interaction of a particle with an obstacle is realized in two stages [9, 10], the first of which is characterized by the shock pressure $p_{\mathrm{sh.w}} = \omega\gamma V$ and the second by the so-called head pressure $p_{\mathrm{f}} = \rho' V^2/2$ with which the particle acts on the contact area. Obviously this should influence the plastic deformation of the obstacle. Because of this, we will assume that the similarity criteria are related by the following relation:

$$W = f\left(\frac{\rho' V^2}{2\sigma}, \frac{\gamma\rho a V}{\sigma}, c_{\mathrm{s}} d^3, \frac{T}{T_{\mathrm{m}}}, \alpha, \frac{\rho}{\rho'}, \frac{a}{a'}\right). \tag{4}$$

The temperature of materials influences their physical properties, especially their strength. Analysis of the dependence of the ultimate strength of materials upon extension σ_{time} and their yield strength upon torsion τ_{t} on the temperature [11] shows that this dependence can be approximated by the expression

$$\sigma = \sigma_0 \cos\theta_T, \quad \theta_T = \frac{\pi}{2}\frac{T - T_0}{T_{\mathrm{m}} - T_0}. \tag{5}$$

As the strength characteristic σ, we used the shear strength, which is practically equal to the ultimate strength in the case of torsion τ_{t}.

The criterion $c_{\mathrm{s}} d^3$ was not verified since, in none of the experiments analyzed by us was the density of the particle flux impacting the obstacle indicated. If the particle size substantially influences the erosion rate, points with a different d should be far apart from each other.

The initial experimental data on the erosion of materials by particles impacting them at a right angle were taken from [12–15] and processed with the use of similarity criteria. It was assumed that dependence (4) has the exponential form

$$W = C\left(\frac{\rho' V^2}{2\tau_{\mathrm{t0}}\cos\theta_T}\right)^n \left(\frac{\gamma\rho a V}{\tau_{\mathrm{t0}}\cos\theta_T}\right)^m \left(\frac{\rho}{\rho'}\right)^l \left(\frac{a}{a'}\right)^k. \tag{6}$$

It has been established using the least-squares technique that $C = 1.41\cdot10^{-5}$, $n = 0.475$, $m = 1.570$, $l = 1.755$, and $k \approx 0$. The root-mean-square error of the formula was 34%.

We also considered the possibility of generalization of experimental data in the form of a functional dependence similar to the relation presented in [16]:

$$W = \frac{V^2}{2H_{\mathrm{er}}} f\left(\mathrm{M}, \frac{T}{T_{\mathrm{m}}}, \frac{\rho}{\rho'}\right), \quad \mathrm{M} = \frac{V}{a}. \tag{7}$$

The erosion enthalpy H_{er}, in the opinion of Yu. V. Polezhaev and D. S. Mikhatulin [16], is a characteristic of the obstacle material, dependent only on its temperature. As a result of the generalization of the same experimental data, the following criteria formula has been obtained:

$$W = \frac{V^2}{2H_0}\frac{\mathrm{M}^{0.6}}{\cos^{1.5}\theta_T}\left(\frac{\rho}{\rho'}\right)^{0.75}. \tag{8}$$

The quantity H_0 having the dimension of specific energy was found to be constant and equal to $1.5\cdot10^6$ J/kg. The root-mean-square error of this formula is somewhat lower and is equal to 30%. Generalization of the entire set of data in the form of (3) has given a root-mean-square error of 85% at $m = 1.9$ (Fig. 1a).

Figure 1b shows a comparison of the dependence calculated by formula (8) with the data of experiments where

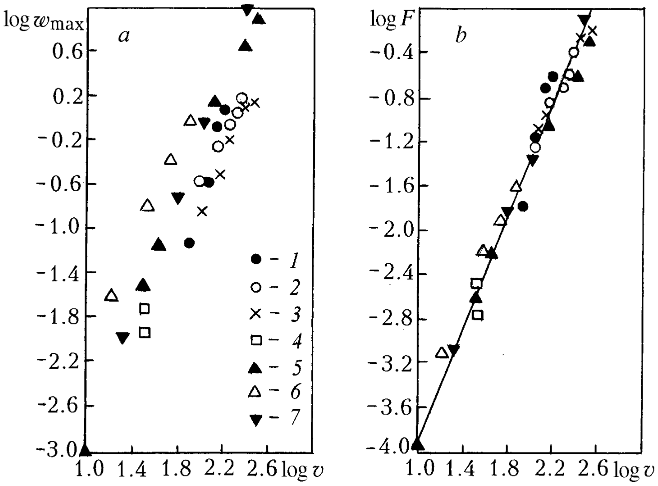

Fig. 1. Processing of experimental data by formula (3) (a) and formula (8) (b): 1) Al_2O_3 (particles) — Al (obstacle) [12]; 2) glass — Al [13]; 3) steel — Al [13]; 4) SiO_2 — Al [15]; 5) SiO_2 — steel [15]; 6) SiO_2 — steel [14]; 7) Al_2O_3 — Cu [15].

$$F = \left[\frac{V^{2.6}}{2H_0}\right] = Wa^{0.6}\cos^{1.8}\theta_T\left(\frac{\rho}{\rho'}\right)^{-0.76} .$$

The function F is equal to the expression in brackets if formula (8) is exact. This dependence is shown by the solid line in Fig. 1b. When Fig. 1a is compared with Fig. 1b, it is apparent that expression (8) generalizes the experimental data fairly well. One more important conclusion is that the erosion rate is independent of the particle sizes.

Clearly the set of experimental data is insufficiently representative since it describes only 30 points; therefore, it is necessary to use additional data to verify the criteria formulas. Nonetheless, we would expect that this set will not be substantially changed since the parameters used in the experiments analyzed were varied in fairly large ranges. For example, the velocity of the particles was changed from 10 to 300 m/sec, the temperature of the obstacle varied from room temperature to 0.8 T_m, the density ratio changed from 0.3 to 3, and the particle diameter varied from 40 to 3000 μm.

It is known that the rate of dusting of plastic materials depends on the angle α at which particles are incident on their surface. From the maximum-breakdown standpoint, the optimum angle of incidence of particles on metals lies between 20 and 50^o depending on the properties of the obstacle material. The angle of incidence can be determined from the formula

$$W(\alpha) = W(90^o)\, g\,(w_{\max}, \alpha)\,, \tag{9}$$

where $W(90^o)$ is calculated by the above formulas and the function $g(w_{\max}, \alpha)$ depends on the ratio of the maximum erosion rate to the erosion rate at a right angle of incidence of particles $w_{\max}$ and on the angle of incidence α. To determine the function $g(w_{\max}, \alpha)$, we approximated the data of [12] on the erosion of aluminum at different temperatures by particles incident on its surface at different angles in the form

$$g\,(w_{\max}, \alpha) = \frac{w_{\max}\alpha}{\alpha_n\left(\dfrac{\alpha - \alpha_{\max}}{90^o - \alpha_{\max}}\right)(w_{\max} - 1) + \alpha} . \tag{10}$$

Analysis has shown that the value of $w_{\max}$ depends linearly on the ratio between the temperature of the material and its melting temperature:

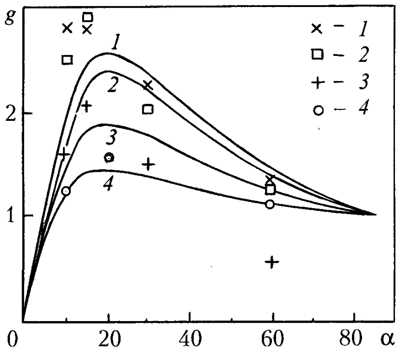

Fig. 2. Comparison of the data calculated by formula (10) with the experimental data: 1) T/T_m = 0.32; 2) 0.4; 3) 0.6; 4) 0.8.

$$w_{\max} = 3.25 - 2.25\frac{T - T_0}{T_m - T_0}\,. \tag{11}$$

The root-mean-square error of the calculations by formula (10) does not exceed the error of formulas (6) and (8).

Figure 2 shows a comparison of the experimental data of [12] with the data calculated by formula (10). At a melting temperature $g(w_{\max}, \alpha) \equiv 1$ and the erosion rate calculated for the case of incidence of particles at a right angle, $W(90^0)$ tends to infinity, which points to a change in the character of breakdown of the material.

Thus, we have obtained criteria relations that allow one to predict the rate of dusting of materials by particles impacting them. This makes it possible to optimize technological processes based on the dusting effect.

NOTATION

a, velocity of sound, m/sec; B, hardness of the obstacle material, N/m^2; b, coefficient in formula (3); c_s, concentration of particles in the flux, m^{-3}; C, coefficient in formula (6); d, particle diameter, m; E, erosion resistance of the obstacle material, kg·m^{-1}·sec^{-2}; F, function (see Fig. 1b); g, function in formula (9); H, hardness of the particle material, N/m^2; H_0, constant in formula (8), N/m^2; H_{er}, erosion enthalpy in formula (7), J/kg; i, number of independent variables; M, Mach number; m, exponent in formula (3); k, l, m, n, exponents in formula (6); N, number of determining similarity criteria; p, load, pressure, N/m^2; R, effective curvature of the particles; T, temperature of the obstacle, K; T_m, melting temperature, K; V, velocity of the particles, m/sec; W, erosion rate; $w_{\max}$, maximum erosion rate related to the erosion rate in the case of impact of particles upon an obstacle at a right angle; z, number of primary dimensions; α, angle of incidence of particles, deg; γ, function of the impedances of the particles and the obstacle; ν, Poisson ratio; θ_T, temperature parameter in formula (5); ρ, density, kg/m^3; σ, characteristic of the obstacle strength, N/m^2; σ_{time}, time breaking strength, N/m^2; τ_t, yield strength in the case of torsion, N/m^2; τ_{cr}, critical shear stress, N/m^2; ω, acoustic impedance, kg/(m^2·sec). Subscripts: time, time; cr, critical state; er, erosion; f, head; max, maximum; m, melting; s, solid; sh. w, shock wave; t, torsion; T, temperature; 0, normal conditions; $'$, particle.

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